Supplemental Material

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1 A Technical Lemma

Lemma 2: Let us consider the matrices $\mathbf{F}$ and $\mathbf{A}$ defined in Eqs. (8) and (9) of the main paper, respectively. Then, the following relation holds

$$rank\{\mathbf{A}\} \leq rank\{\mathbf{F}\}$$

with the equality holding if and only if the following K constraints hold:

$$rank(\mathbf{M}_k) = rank(\hat{\mathbf{P}}_k) = 1,\ k = 1, \cdots, K. \tag{1}$$

Proof: Let us consider matrices of the aforementioned form:

$$\mathbf{F} = \sum_{k=1}^{K} \lambda_k \mathbf{F}_k = \sum_{k=1}^{K} \lambda_k (\mathbf{M}_k \otimes \hat{\mathbf{P}}_k).$$

Then, using the mutual exclusiveness of the matrices $\mathbf{F}_k,\ k = 1, \cdots, K$, that is:

$$\mathbf{F}_l \odot \mathbf{F}_m = \mathbf{0},\ l \neq m = 1, \cdots, K,$$

where $\odot$ denotes the element-wise product operator, and the well known equality:

$$\begin{aligned} rank(\mathbf{F}_k) &= rank(\mathbf{M}_k \otimes \hat{\mathbf{P}}_k) \\ &= rank(\mathbf{M}_k) rank(\hat{\mathbf{P}}_k), k = 1, \cdots, K, \end{aligned}$$

the following equality holds:

$$
\begin{aligned}
K_{\mathbf{F}} &= rank(\mathbf{F}) \\
&= rank(\sum_{k=1}^{K} \lambda_k \mathbf{F}_k) \\
&= \sum_{k=1}^{K} rank(\mathbf{F}_k) \\
&= \sum_{k=1}^{K} rank(\mathbf{M}_k) rank(\hat{\mathbf{P}}_k).
\end{aligned}
$$

Let us now consider the matrix $\mathbf{A}$ that constitutes a rearrangement of the matrix $\mathbf{F}$, i.e.:

$$
\mathbf{A} = \sum_{k=1}^{K} \lambda_k \mathbf{m}_k \mathbf{p}_k^t = \sum_{k=1}^{K} \lambda_k \mathbf{A}_k
$$

where $\mathbf{m}_k,\ \mathbf{p}_k,\ k = 1, \cdots, K$ are the column-wise vectorized forms of matrices $\mathbf{M}_k,\ \hat{\mathbf{P}}_k,\ k = 1, \cdots, K$ respectively. Then, since matrices $\mathbf{A}_k,\ k = 1, \cdots, K$ are mutually exclusive, it is clear that:

$$
rank(\mathbf{A}) = K.
$$

Note that $K_{\mathbf{F}}$ achieves its minimum value, i.e. K, when the K constraints (1) hold and this concludes the proof of the lemma. □

2 A toy example

One toy example of a façade based on the model of Eq. (3) in the main paper with:

$$
\begin{aligned}
\mathbf{P}_k &= \mathbf{p}_k \mathbf{p}_k^T,\ k = 1,\ 2, \text{ and} \\
\mathbf{P}_3 &= \mathbf{p}_3 \tilde{\mathbf{p}}_3^T \qquad (2) \\
\mathbf{P}_4 &= \mathbf{P}_1, \qquad (3)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{p}_1 &= [\mathbf{0}_{1\times 25}\ \mathbf{1}_{1\times 50}\ \mathbf{0}_{1\times 25}]^T \\
\mathbf{p}_2 &= [\mathbf{0}_{1\times 10}\ \mathbf{1}_{1\times 30}\ \mathbf{0}_{1\times 20}\ \mathbf{1}_{1\times 30}\ \mathbf{0}_{1\times 10}]^T \\
\mathbf{p}_3 &= [\mathbf{0}_{1\times 35}\ \mathbf{1}_{1\times 30}\ \mathbf{0}_{1\times 35}]^T \\
\tilde{\mathbf{p}}_3 &= [\mathbf{0}_{1\times 10}\ \mathbf{1}_{1\times 80}\ \mathbf{0}_{1\times 10}]^T \qquad (4)
\end{aligned}
$$

$$
\mathbf{M}_1 = \begin{bmatrix} \mathbf{1}_{3\times 2} & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{2\times 2} & \mathbf{0}_{2\times 1} \end{bmatrix} \quad \mathbf{M}_2 = \begin{bmatrix} \mathbf{0}_{3\times 2} & \mathbf{0}_{3\times 1} \\ \mathbf{1}_{2\times 2} & \mathbf{0}_{2\times 1} \end{bmatrix}
$$
$$
\mathbf{M}_3 = \begin{bmatrix} \mathbf{0}_{3\times 2} & \mathbf{1}_{3\times 1} \\ \mathbf{0}_{2\times 2} & \mathbf{0}_{2\times 1} \end{bmatrix} \quad \mathbf{M}_4 = \begin{bmatrix} \mathbf{0}_{3\times 2} & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{2\times 2} & \mathbf{1}_{2\times 1} \end{bmatrix}, \qquad (5)
$$

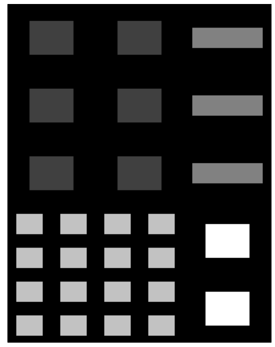


Figure 1: Urban building façade based on the model of Eq. (3) of the main paper with weighting coefficients $\lambda_1 = 40$, $\lambda_2 = 160$, $\lambda_3 = 80$ and $\lambda_4 = 255$.

is shown in Fig. 1. Note that the above defined matrices $\mathbf{M}_k,\ k = 1,\ 2,\ 3$ satisfy Eq. (1) and (2) of the main paper. In addition, as it is clear from Eqs. (3-5), all matrices as well as all patterns are of rank one.

3 Experiment Results

We present additional figures to demonstrate the performance of our method. In each figure, the five columns represent original input image, partitioned blocks, detected low-rank component, detected patterns by Kronecker product model, and ground truth, respectively.

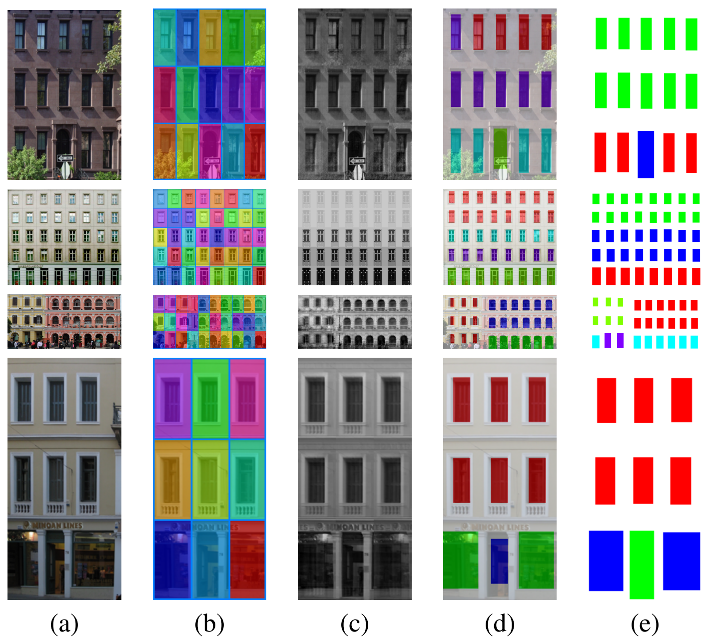


Figure 2: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

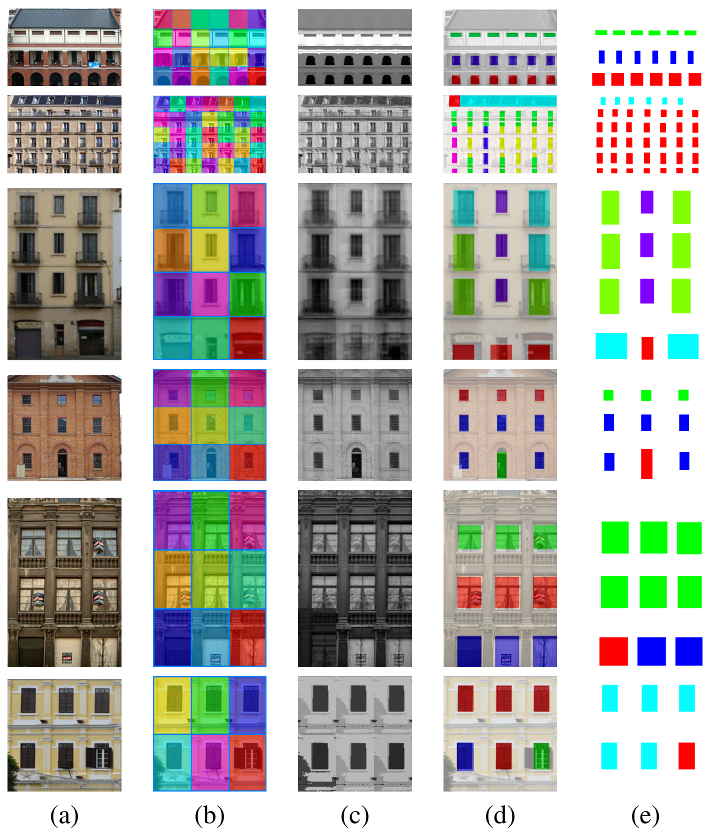
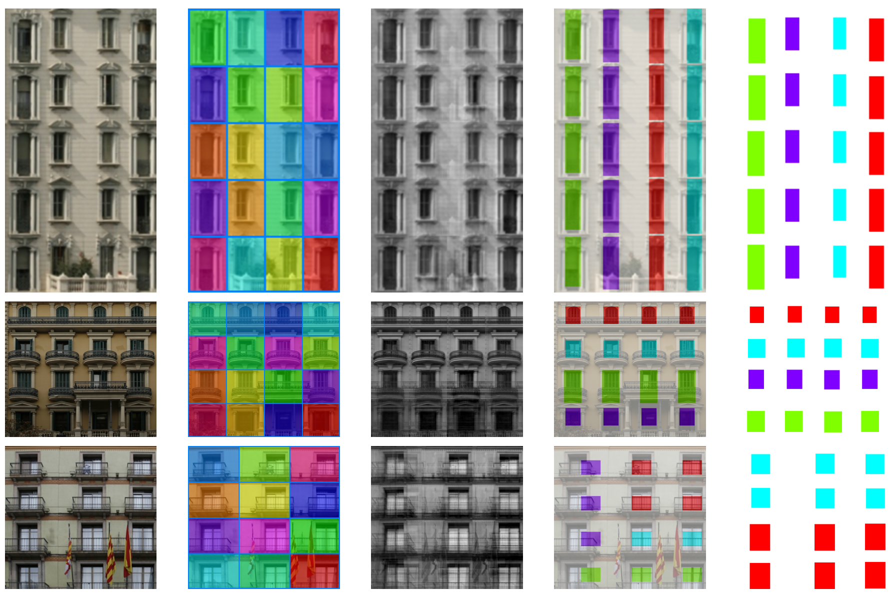


Figure 3: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.



(a) (b) (c) (d) (e)

Figure 4: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

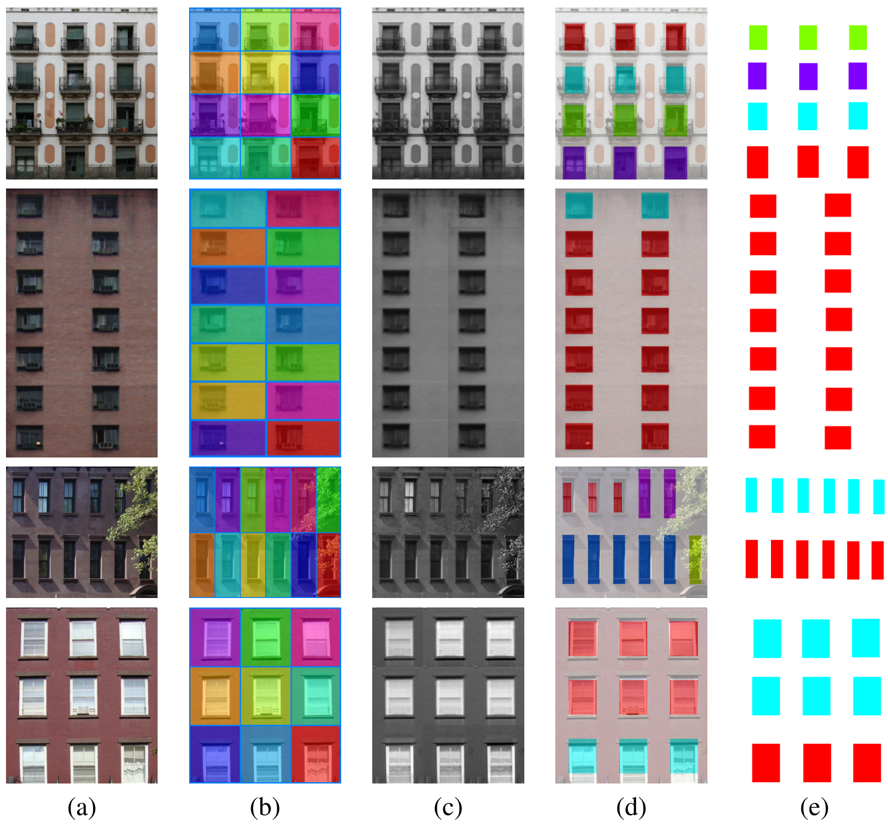


Figure 5: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

(a) (b) (c) (d) (e)

Figure 6: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

(a) (b) (c) (d) (e)

Figure 7: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

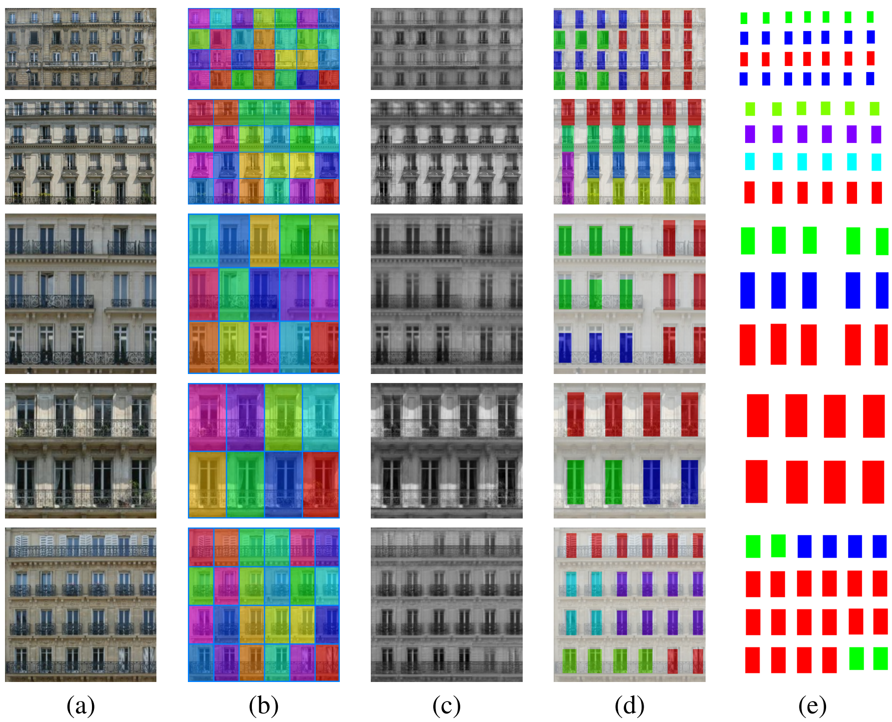


Figure 8: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

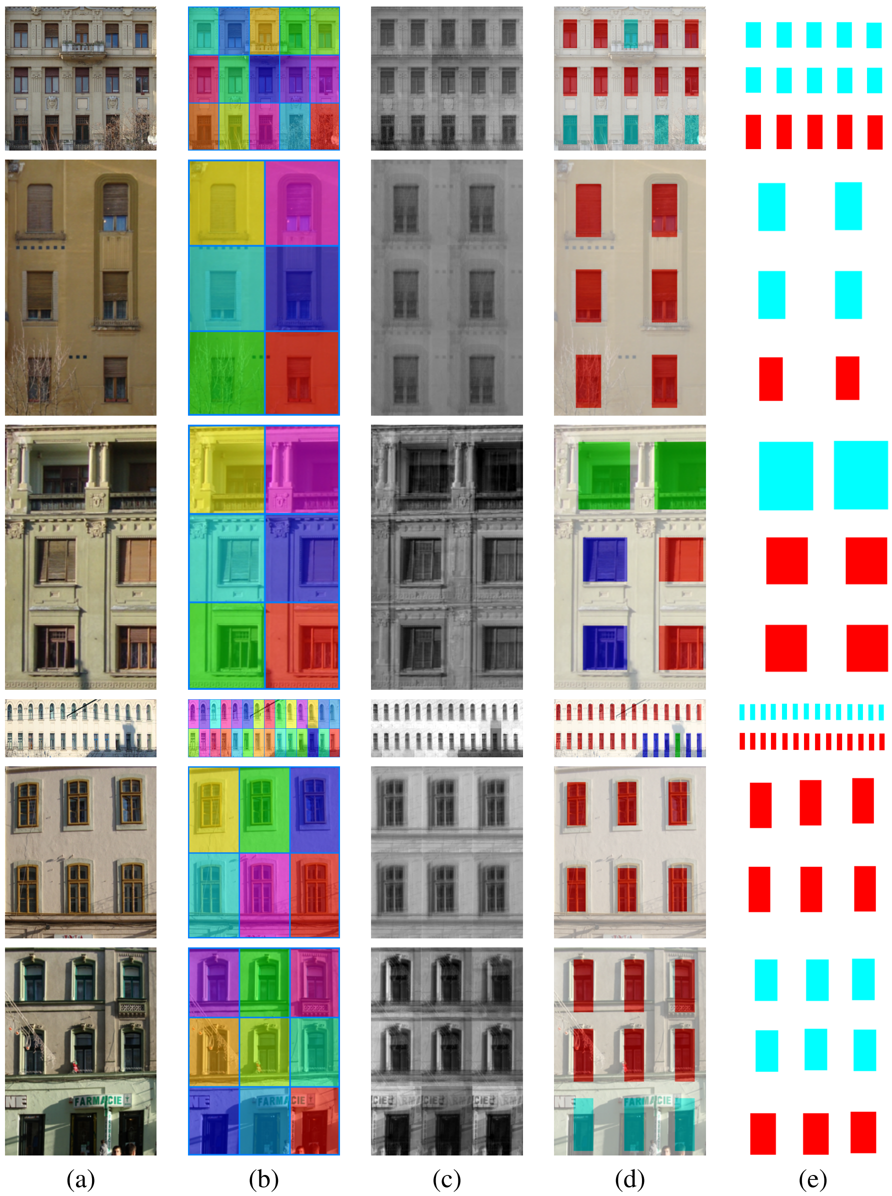


Figure 9: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

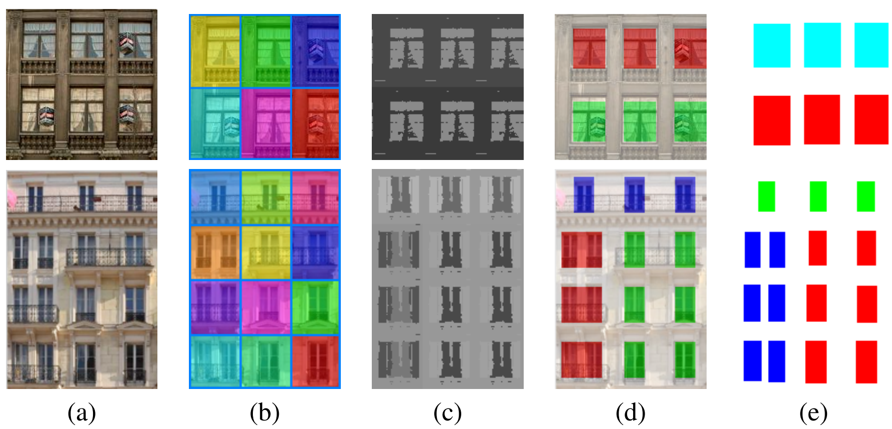
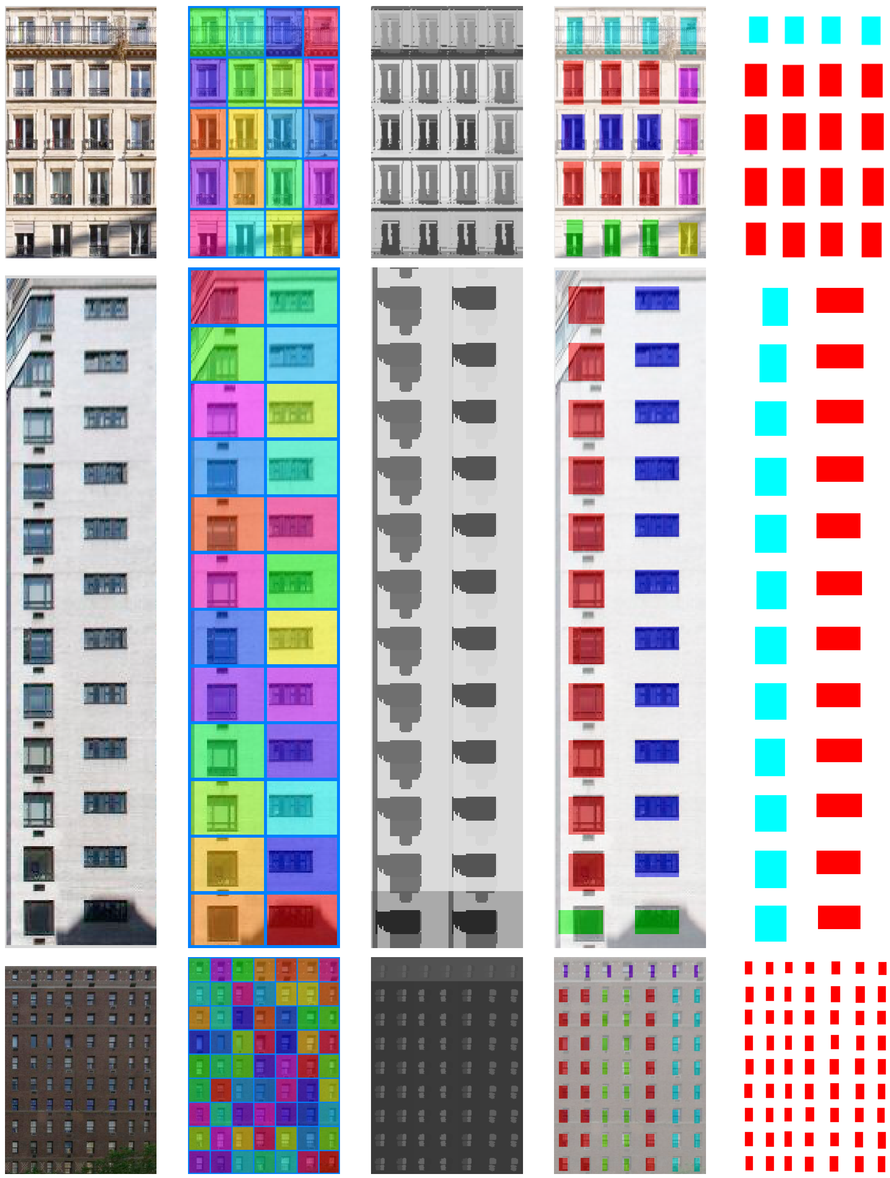


Figure 10: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

Figure 11: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.



(a) (b) (c) (d) (e)

Figure 12: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

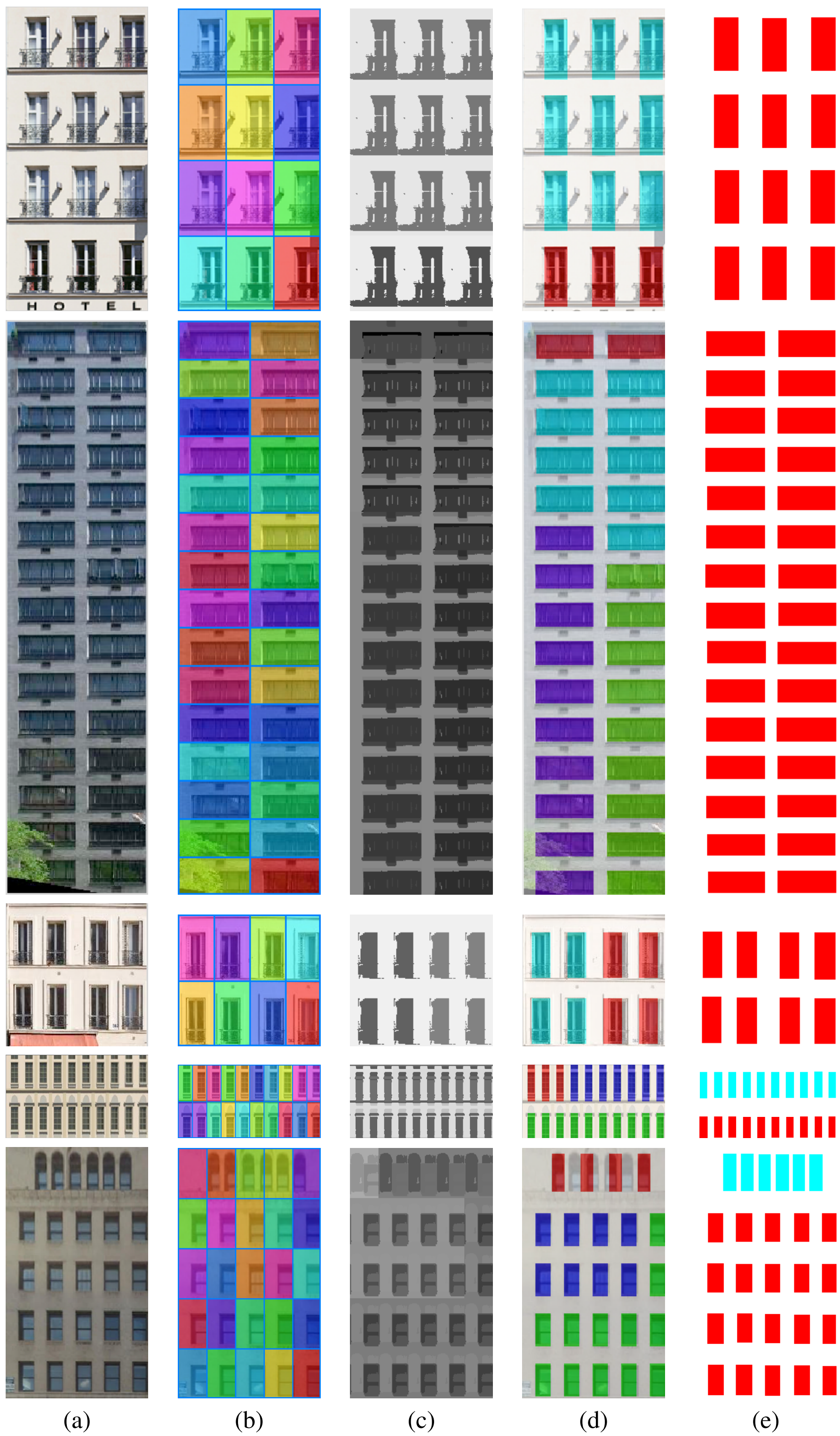
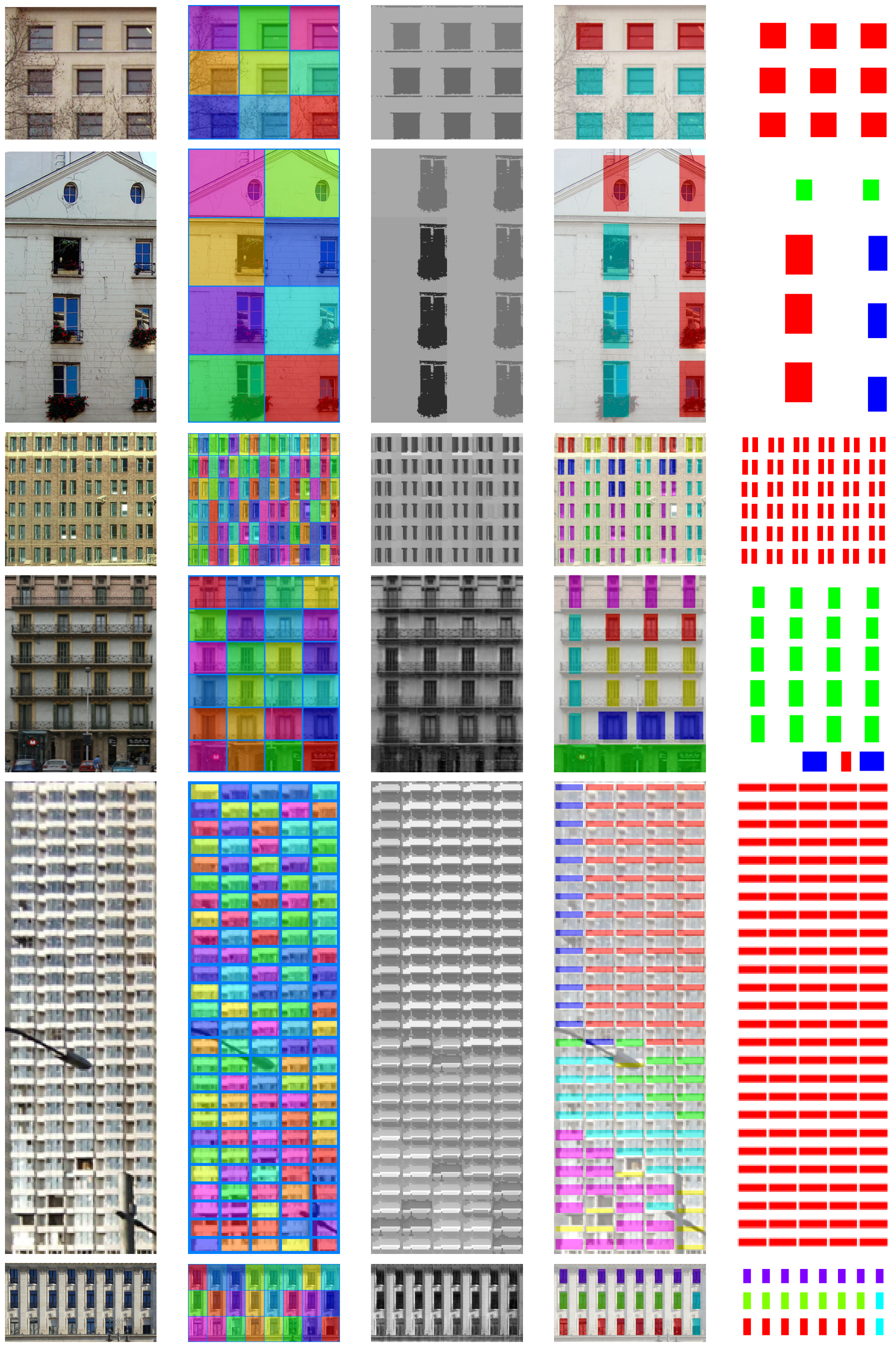


Figure 13: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.



(a) (b) (c) (d) (e)

Figure 14: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

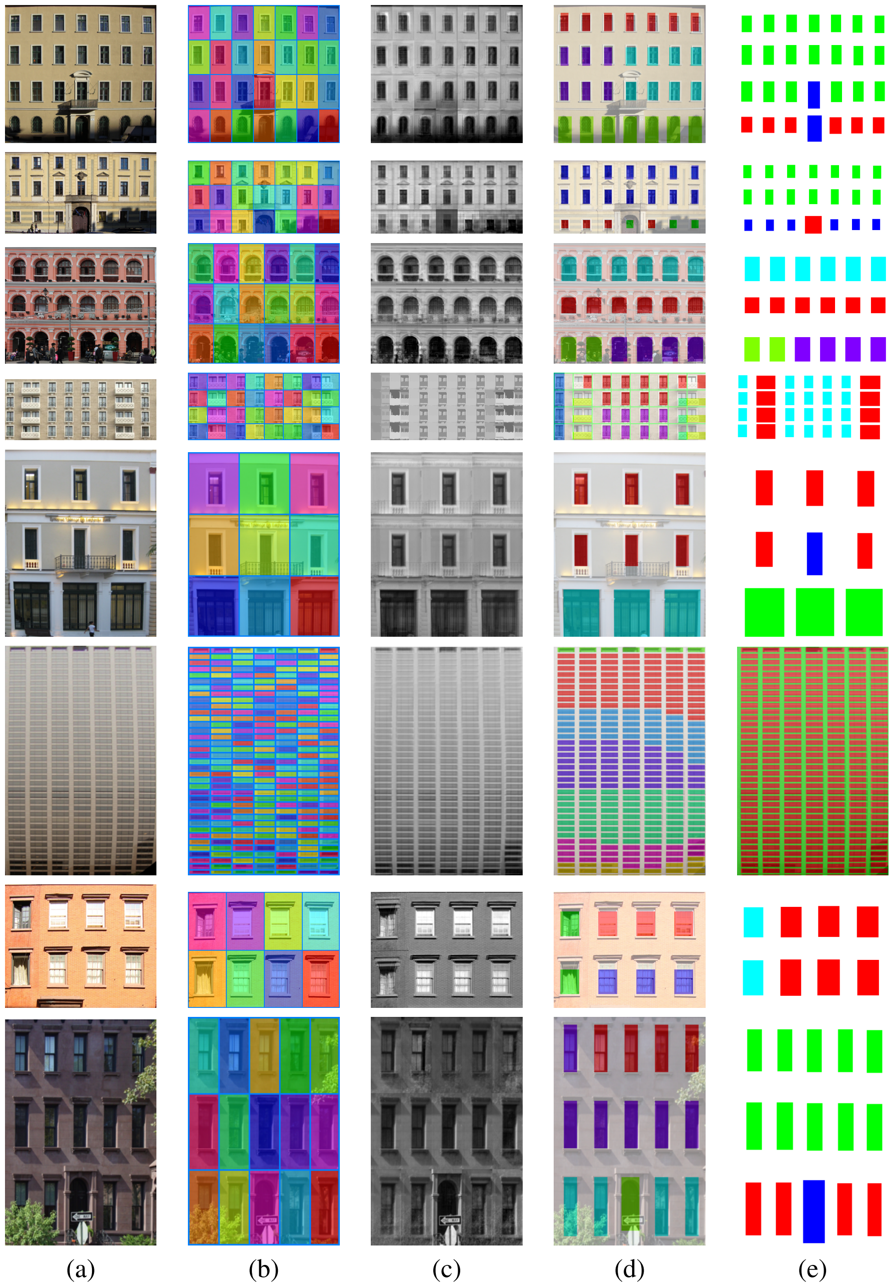


Figure 15: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

(a) (b) (c) (d) (e)

Figure 16: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.

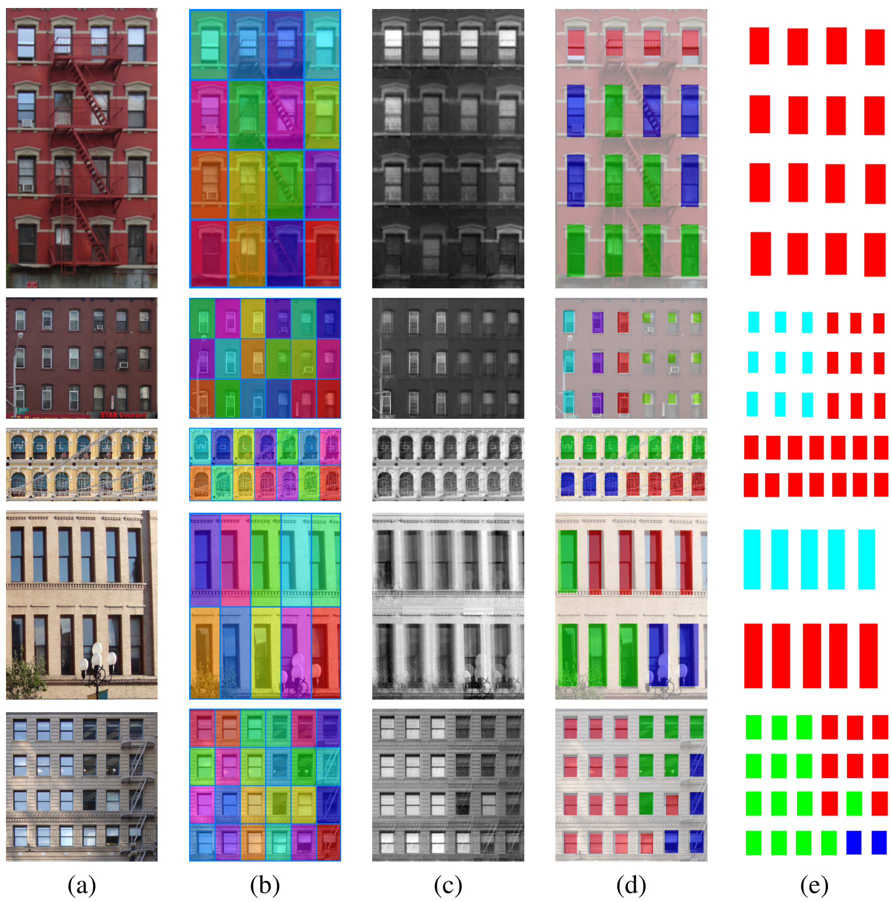
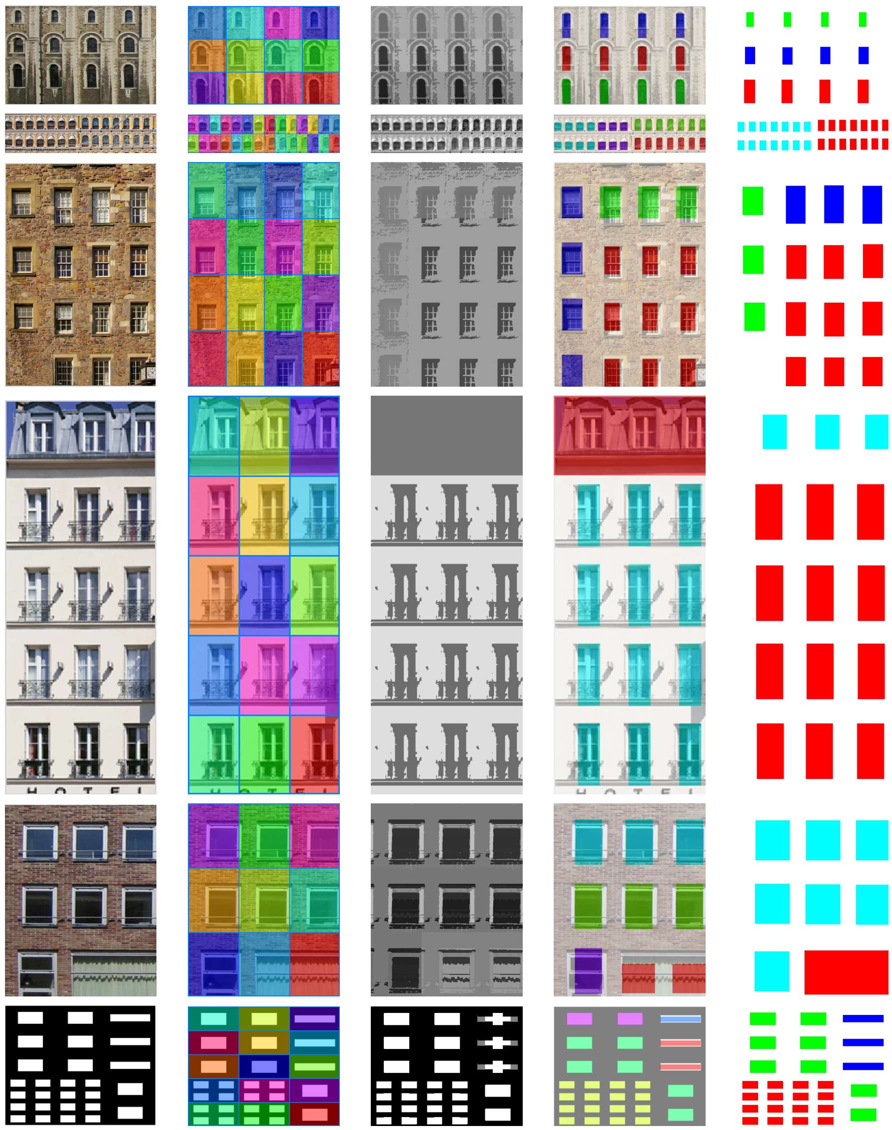


Figure 17: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.



(a) (b) (c) (d) (e)

Figure 18: (a) Input image, (b) detected partitions, (c) low-rank component, (d) detected repeated patterns, and (e) ground truth.